

SPIN-DRIVEN INFLATION

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Abstract

Following recent studies of Ford, we suggest – in the framework of general relativity – an inflationary cosmological model with the self-interacting spinning matter. A generalization of the standard fluid model is discussed and estimates of the physical parameters of the evolution are given.

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In the recent paper of Ford [1] a new inflationary model was proposed in which the inflation mechanism was related to the existence of a self-interacting vector field, replacing the usual scalars. Such a structure could be explained by the nonlinear Lagrangian, *effectively* arising from a more fundamental interactions between the elements of cosmological matter. However, the vector field of a non-gauge nature does not appear to be a physically important object, in particular, not in cosmology. In this paper we use the main idea of Ford and propose an alternative mechanism for inflation, which is related to the spin of matter. Spin-spin interactions play an important role in many field-theoretical models, in particular in torsion theories [2]. We will not specify the form of a possible fundamental spin-spin interaction (although taking the Poincaré gauge approach [3] as the basic model), but instead proceed within a similar *effective* approach in the framework of the spinning fluid variational theory. The crucial point of the proposed generalization is the introduction of a non-linear spin term in the fluid Lagrangian. It represents the contribution of the effectively averaged Poincaré rotational gauge fields, e.g., in the sense of integrating away the torsion [4].

Let us postulate the Lagrangian for the spinning fluid with self-interaction

$$L_m = \epsilon(\rho, s, \mu^{ij}) - \frac{1}{2}\rho\mu^{ij}b_i^\mu(\nabla_\alpha b_j^\nu)u^\alpha g_{\mu\nu} \\ + \rho u^\mu \partial_\mu \lambda_1 + \lambda_2 u^\mu \partial_\mu X + \lambda_3 u^\mu \partial_\mu s + \lambda^{ab}(g_{\mu\nu}b_a^\mu b_b^\nu - \eta_{ab}) + V(\xi). \quad (1)$$

Here the terms with the Lagrange multipliers λ describe the usual constraints [5], imposed on the fluid variables ρ (particle density), s (specific entropy), X (identity Lin coordinate), μ^{ij} (specific spin density), and b_a^μ (material tetrad, with $b_0^\mu = u^\mu$ the 4-velocity of elements). [Indices from the middle of the Latin alphabet $i, j = 1, 2, 3$ refer to the local 3-frame defined by the spacelike vectors of the material tetrad; the Greek indices $\alpha, \beta, \dots = 0, 1, 2, 3$ refer to space-time coordinates]. The variable

$$\xi = \frac{1}{2}\rho^2\mu^{ij}\mu_{ij} \quad (2)$$

is the square of the spin density of the matter. The function $V(\xi)$ can be thought as a kind of the effective potential which arises due to the underlying fundamental interactions between particles with spin. A particular example is given by the Einstein-Cartan theory [2] in which V is a linear function of ξ .

The equations of motion of the fluid and the gravitational Einstein equations are derived from the variation of (1) with respect to the fluid and gravitational variables. This yields the following modified energy-momentum tensor of the spinning fluid

$$T_{\mu\nu} = \epsilon u_\mu u_\nu - p h_{\mu\nu} - 2(g^{\alpha\beta} + u^\alpha u^\beta)\nabla_\alpha(u_{(\mu}S_{\nu)\beta}) + \\ + g_{\mu\nu}V - h_{\mu\nu}2\xi V', \quad (3)$$

where $(')$ denotes the derivative with respect to ξ , and, as usually, p is the pressure, $S^{\mu\nu} = -\frac{1}{2}\rho b_i^\mu b_j^\nu \mu^{ij}$ is the spin density, and the projector on the subspace orthogonal to 4-velocity is $h_\nu^\mu = \delta_\nu^\mu - u^\mu u_\nu$.

The dynamics of the spin is not changed, and the standard equation of motion is valid (which follows from the variation of (1) with respect to the material tetrad):

$$\nabla_\mu(u^\mu S_{\alpha\beta}) = u_\alpha u^\lambda \nabla_\mu(u^\mu S_{\lambda\beta}) - u_\beta u^\lambda \nabla_\mu(u^\mu S_{\lambda\alpha}). \quad (4)$$

For concreteness, in (1)-(4) we consider the case of the ordinary spinning fluid. Eq.(4) shows that the motion of spin is not affected by the nonlinearity: this is essentially a rotation, precession. The same, however, is true also for the generalized fluid with magnetic moment and electric charge in the electromagnetic field.

But the dynamics of the fluid itself is changed. It is described by the conservation of the energy-momentum, and for (3) this reads

$$\begin{aligned} \nabla_\mu T^\mu_\nu &= u_\nu(u^\mu \nabla_\mu \epsilon + \epsilon \nabla_\mu u^\mu + p \nabla_\mu u^\mu) \\ -h^\mu_\nu \nabla_\mu(p + 2\xi V') &+ (p + \epsilon + 2\xi V')a_\nu + \nabla_\nu V + u_\nu(\nabla_\mu u^\mu)2\xi V' \\ &+ 2u^\mu S_{\nu\lambda} \nabla_\mu a^\lambda + R_{\alpha\beta\mu\nu} u^\mu S^{\alpha\beta} = 0. \end{aligned} \quad (5)$$

Standard projections on the u^μ and the orthogonal directions yield

$$(p + \epsilon) \nabla_\mu u^\mu + u^\mu \nabla_\mu \epsilon = 0, \quad (6)$$

$$(p + \epsilon + 2\xi V')a_\nu - h^\mu_\nu \nabla_\mu(p + 2\xi V' - V) + 2S_{\nu\mu} u^\lambda \nabla_\lambda a^\mu + R_{\alpha\beta\mu\nu} u^\mu S^{\alpha\beta} = 0. \quad (7)$$

Now we are in a position to proceed in a way typical for the inflational approach: it is necessary to specify the “effective potential” so that the inflationary stage becomes possible.

It is straightforward to analyse the modification of the standard cosmologies. Let us consider the special case of a flat model with the line element

$$ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2).$$

With the self-interacting spinning fluid (3), the Einstein equations become

$$3\frac{\dot{R}^2}{R^2} = \kappa(\epsilon + V), \quad (8)$$

$$-(2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}) = \kappa(p - V + 2\xi V'). \quad (9)$$

Instead of (9) it is more convenient to use the conservation law (6), which reads as usual:

$$\dot{\epsilon} + 3\frac{\dot{R}}{R}(\epsilon + p) = 0. \quad (10)$$

Clearly, eqs. (8) and (10) must be supplemented by the equation of state $p = p(\epsilon)$. Notice that eq. (9) follows from (8) and (10).

It is important to note the specific behaviour of the argument ξ of the potential. It is the square of the spin ($\xi = 2S_{\mu\nu}S^{\mu\nu}$), and from the equations of motion (4) one finds

$$\xi(R(t)) = \frac{const}{R^6}. \quad (11)$$

Thus, during the expansion of the universe,

$$V(\xi) \rightarrow V(0).$$

Following the lines of reasoning of Ford (who studied inflation with a vector field), one can discuss now the possible form of the potential V . Most appropriate seems to be what is called in [1] the “chaotic inflation” model, in which inflation occurs for large ξ and reheating as $\xi \rightarrow 0$. A potential which provides such a possibility reads

$$V(\xi) = V_0(1 - e^{-\alpha\xi}), \quad (12)$$

where the parameters V_0, α are chosen from the estimates for the characteristics of the inflationary period.

It is clear from (8) that the cosmological evolution — the dynamics of the scale factor $R(t)$ — depends on the relative value of the ϵ and V . Namely, when V is much smaller than the energy density, the evolution proceeds as in the standard cosmology. For (12) this occurs inevitably as ξ tends to zero. However, if ϵ is less than V and the latter is approximately constant, the inflation occurs. Let us for definiteness choose the equation of state $p = \frac{\epsilon}{3}$. Then from (10)

$$\epsilon = \frac{const}{R^4}. \quad (13)$$

Clearly, at the initial stages (when $R \sim 0$) the energy density is greater than the constant V_0 (which is the value of the spin potential at early times, when ξ is very large). We can roughly consider the moment when ϵ becomes equal V_0 to be the starting point of inflation. Let us denote the values of the energy and spin density at that moment as

$$\epsilon_0, \quad S_0.$$

As the end of the inflationary stage (approximately) could be considered the point $t = \tau$ at which the (decreasing) energy density $\epsilon(\tau)$ again becomes equal to some $V(\xi(\tau))$ (which is decreasing much more fastly). Let us denote the ratio of the scale factors at the beginning and the end of the inflationary stage by

$$E = \frac{R_0}{R(\tau)}. \quad (14)$$

One has $\xi(\tau) = \xi_0 E^6 = S_0^2 E^6$, $\epsilon(\tau) = \epsilon_0 E^4$. Hence from (12) one can derive the relation between these quantities:

$$\epsilon(\tau) = V(\tau) \quad \rightarrow \quad E^4 = 1 - e^{-\alpha S_0^2 E^6} \approx \alpha S_0^2 E^6, \quad (15)$$

Assuming, that $E^{-1} \approx 10^{20}$, this gives the estimate for the constant α and the initial spin density,

$$\alpha S_0^2 \approx 10^{40}. \quad (16)$$

This is large enough, so at the beginning of the inflation V is constant, equal to V_0 . Very crudely let us assume that $V_0 = 10^{94} \frac{g}{cm^3}$ ($= \epsilon_0$), i.e. inflation starts directly at Planckian values. Then at the end of inflation $\epsilon(\tau) = 10^{14} \frac{g}{cm^3}$ which is about the quark density. The duration of the inflationary stage is then of order of a Planckian time, estimated by

$$\tau = \frac{20 \log 10}{\sqrt{\frac{\kappa V_0}{3}}} \approx 6 \times 10^{-43} s. \quad (17)$$

Thus in this scheme, analogously to the developments of Ford one can predict an inflation stage during the universe's evolution, which is caused by a non-linear effective potential arising from a fundamental interaction of the cosmological matter. In our opinion, the spin non-linearity is much more natural than the vector field one, originally used in [1].

Let us point out that one can also develop the original idea of Ford into a kind of general scheme: it is clear, that the crucial point is to have some self-interacting (i.e. non-linear) system coupled to the Einstein gravity, then for special choices of a "potential" (non-linearity) one can discover inflation.

The estimates derived above can be changed and improved for the potentials different from (12). An interesting problem is to derive the form of the spin nonlinearity V when the fluid is considered as a semiclassical description of a realistic quantum matter.

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